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On the modeling of the new student acceptance status through science and technology written test using bernoulli mixture model

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Abstract. This research aimed to model StudentAcceptance Status at the Sepuluh Nopember Institute of Technology (ITS) through the written test of science and technology, using Bernoulli Mixture Model in order to evaluate the new student acceptance status. BMM distribution was established based on the comparisonbetween the students' scores of the basic abilities, namelyMathematics, Physics, Chemistry, and Biology which corresponded to the majors they had chosen, combined with the Student Acceptance Status (0 and 1). This combination generated two components of Mixture, namely right or wrong. The characteristics of each component were then identified through BMM by involving the covariates of Student Acceptance Status, namely the basic ability test and the scholastic test. The combination of Markov Chain Monte Carlo with the Gibbs Sampling algorithm was employed to estimate the parameters used in this research. This method was applied to the data of prospective students who registered in ITS through written test of science and technology. This research result showed the estimated parameters and the formed model of BMM.

1. Introduction

Finite mixture models are increasingly used in distribution modeling in a variety of random phenomena datasets. The mixture model is one particular model that can accommodate multimodal characteristics. Multimodal characteristic data is data consisting of several subpopulations, in which each subpopulation is a component of the population arranged in a mixture with varying proportions. Finite mixture models are one of the increasingly popular modeling methods due to the flexibility in data modeling and clustering [1]. Besides, such kind of models can provide good results in modeling the heterogeneity of a population by re-conceptualizing subpopulations into more homogeneous groups. Bernoulli Mixture Model (BMM) is the developed mixture modeling of binary data that was first conducted by [2] in 1991 and has been developed rapidly in various fields such as, mainly, text mining [3], handwritten digits recognition [4,5] and microbiology and human genetics [6]. Also, it can be useful in robotics and mechanical engineering as an upgrade in the camera sensor so that the robots can travels along with the environment [7]. In the research done by [8], BMM helped them in clustering sparse datasets.

In this study, the researchers used the BMM method to model the data of new students who registered in ITS through a written test, expecting that the modeling results of this study can give a contribution to the evaluation and improvement of the classical scoring system, which had been focusing

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only on total value. There had been no change occurring over the years except in the types of questions and the method of taking scores. This condition has caused tutoring agencies to "manipulate" the total score, namely by working on questions the participants considered them easy, which caused themto focus only on the subject they are good at and ignore the others they find them difficult. For example, there were students A and B wanted to enter the statistics department. The total score of student A was 400, while that of student B was 360.But, when we check their scoresfor each subject, we find out that student A is not quite acceptable to be enrolled in statistics (since his math and basic math score are low) and student B is not good at English but good at math and basic math whose scoresare correspondent with the program he chose. Fig.1, shows that some students who are not accepted the program study have higher scores than those accepted.

We tried to model the admission of new students, but the acceptance required the score taken from the basic ability test results, which corresponded to the study programs chosen by the students, namely score of Mathematics, Natural Sciences, Chemistry, Physics, and Biology. These requirements were called Mixture. The BMM method was chosen for its better accuracy in classification than the binary logistic regression method [9]. Based on research conducted by [10] suggested that the Bernoulli Mixture Method algorithm is considered very simple but providing accurate and powerful results. The mixtures used in this study were a comparison between the basic ability test scores that corresponded to the study programs chosen by prospective students. The characteristics of each component of Bernoulli mixtures can be explained by the results of BMM modeling by including variables that affectedStudent Acceptance Status, which consisted of six covariate variables, namely three scores of the scholastic test (Fig. 1), including English, Bahasa Indonesian, and Basic Mathematics.

2. Literature Review

2.1. Link Function

Bernoulli distribution with mean p is a discrete random distribution that has only binary results, namely 0 and 1, Therefore, to write p as a linear combination of Equation (1) will not be consistent in the probability law.

$$\eta = \sum_{j=1}^{J} X_j \beta_j \tag{1}$$

Thus, to be adaptable to the probability law, a transformation will be carried out at p, and then called g(p) by mapping intervals (0, 1) to $(-\infty, \infty)$. The existence of this linear relationship causes the distribution of the dependent variable to be transformed into other distributions such as Gaussian, Bernoulli, Poisson, and gamma. Therefore the equation can be written as Equation (2).

$$g(p) = logit(p) = \log\left(\frac{p}{1-p}\right) = X\beta$$
(2)

So that, the probability of *p* can be known from the transformation result link function g(p) [7].

$$p = \frac{\exp(X\beta)}{1 + \exp(X\beta)},\tag{3}$$

2.2. Bernoulli Mixture Model

Bernoulli Mixture model is the expansion of multivariate Bernoulli, which has a Bernoulli distribution that has *K* group (*K* is unknown but considered to have a limit), with *Y* as a random variable that has independent vector components $n, y = [y_1, y_2, ..., y_n]^T$. In which *K* is a finite number that has proportion with $\pi = \pi_1, \pi_2, ..., \pi_K$, the finite mixture model can be indicated as the following equation [9];[12].

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$$(y_i) = \sum_{k=1}^{K} \pi_k p_k(y_i),$$
(4)

in which k is the number of mixture component while $k = 1, 2, ..., K, p_k(y_i)$ is the density function of mixture, and π_k is the mixture proportion that is a non-negative quantity that amounts to 1 based on the following Equation (5)

$$\sum_{k=1}^{K} \pi_k = 1,$$
 (5)

Based on Equation (1), BMM is aimed at partitioning random variable Y into K groups (depending on the researchers, but finite), so the density of finite mixture model follows (6);[13]

$$P(Y|k,\pi,\Theta) = \sum_{k=1}^{K} \pi_k p_k \left(Y|\Theta_k \right), \tag{6}$$

in which $p_k(.)$ is a density function in the group k while Θ is a mixture density component parameter with $\Theta = \theta_1, \theta_2, ..., \theta_k$ and π_k is mixture proportion with $\pi = \pi_1, \pi_2, ..., \pi_K$. For k = 2, the BMM model follows (7).

$$P_{mix}(y|\pi, x, \beta) = \pi_1 p_1(y|x, \beta_1) + \pi_2 p_2(y|x, \beta_2)$$
(7)

As the
$$p_1(y|x, \beta_1) = \frac{\exp(\sum x\beta_1)}{1 + \exp(\sum x\beta_1)}$$
 and $p_2(y|x, \beta_2) = \frac{\exp(\sum x\beta_2)}{1 + \exp(\sum x\beta_2)}$

in which β_1 is the component for Mixture 1 and β_2 is the component for Mixture 2

2.3. Bayesian Markov Chain Monte Carlo (MCMC)

The basic idea of MCMC is to generate sample data from the posterior distribution according to the Markov chain process with iterative Monte Carlo iteration to obtain convergent conditions. This MCMC method is widely used in Bayesian analysis because, during the process of obtaining a posterior distribution, Bayesian analysis involves complex and complicated integral processes. One of important features of the MCMC method is that it can reduce complexity with large dimensions to simpler pieces [14]. The posterior distribution can be obtained through the following process: [15]

- 1. Determining the initial value for each parameter $\theta^{(0)}$
- 2. Reaching the distribution equilibrium condition by means of a sample of T, where the values of t = 1, 2, 3, ..., T, raise the values $\theta^{(t)}$.
- 3. Checking convergence conditions, if the convergence has not been reached, it is necessary to generate more samples.
- 4. Processing the removal or disposal of the first B sample. This process is called the Burn-in period that aims to eliminate the influence of using the initial value.
- 5. Using $\{\theta^{(B+1)}, \theta^{(B+2)}, \dots, \theta^{(T)}\}$ as samples in posterior analysis while checking the convergence of sample data.
- 6. Making a posterior distribution plot: obtain a summary of the posterior distribution (mean, median, standard deviation, quintiles, and autocorrelation)

3. Methodology

3.1. Data Source

The data sources used in this study were secondary data obtained from SNMPTN and SBMPTN Evaluation and Development Work Program year 2018 consisting of 1848 observations in three different study programs. Details of observations in each study program are shown in Table 1.

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Study Program	Registered Student	Accepted Student	
Mathematics	477	51	
Statistics	524	44	
Actuarial Science	847	27	
Total	1848	122	

Table 1. Data Sources.

3.2. Research Variables

The variables used in this study consisted of two groups of test scores, namely scholastic test and basic ability test, in which each group had several constituent values as shown inTable 2.

Var	Description	Data Scale	Notes
Y	Student Acceptance Status	Categorical	1 = Accepted 0 = Not Accepted
X ₁	Score of Figural examination	Ratio	
X ₂	Score of Numerical examination	Ratio	
X ₃	Score of Verbal examination	Ratio	
X4	Score of English language	Ratio	
X ₅	Score of Indonesian language	Ratio	
X6	Score of Basic Mathematics	Ratio	

Table 2.	Research	Variable for	Analysis.
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3.3. Research Design

The modeling process using Bernoulli Mixture Model would follow the research design in Figure 1.

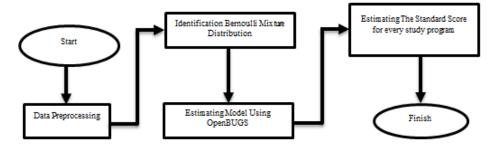


Figure 1. Research Design for Modelling Bernoulli Mixture Model.

4. Results of Study

4.1. Data Preprocessing

The main stages in the BMM process are the identification of Bernoulli mixture distribution. This identification process was carried out in stages by the researchers according to the following steps:

- 1. Setting up the response variable (Y). In this study, the response was Student AcceptanceStatus.
- 2. Selecting the last four Covariates, namely four basic ability test scores in Biology, Physics, Chemistry, and Mathematics.
- 3. Creating a new variable by comparing the basic ability test scores corresponding to the chosen study program. To simplify this step, it was better to group data based on the study program. The new variable in this step was named variable *P*.

4. Obtaining the new variable *P* by coding the basic ability test scores with the following criteria: 0 = if the basic ability test scores corresponding to the chosen study program is not greater than the other basic ability test scores

1 = if the basic ability test scores corresponding to the chosen study program is a greater than the other basic ability test scores

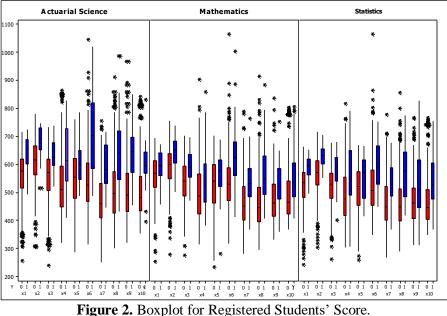
- 5. Determining the right or wrong condition by matching the acceptance status with the variable P. The condition is said to be wrong if the student is accepted but having low basic ability test scores or, on the contrary, the student is not accepted but having the highest basic ability test scores. The condition (Variable AC) is said to be correct if the student is accepted and obtains the highest basic ability test scores and, on the contrary, is said to be wrong when the students is not accepted lowest basic ability test scores.
- 6. Grouping the right andwrong conditions as Mixture 1 and Mixture 2, respectively. See Table 3 for an intensive explanation.

Status	Р	Р	
Status	Not Highest Score (0)	Highest Score (1)	
Not Accepted (0)	Right	Wrong	
Accepted (1)	Wrong	Right	

Table 3.	Identification	of Bernoulli Mixture.
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4.2. Student Characteristics

Based on Table 1, we can see that 1848 students applied in three study programs, namely Mathematics, Statistics and Actuarial Science. Out of those, only 122 or 6.6% were accepted. Only 51 of 477 applicantswere accepted in Mathematics study program. It means that competitiveness in Mathematicswas 1:9. As for Statistic study program, there were 524 applicants, but only 44 students were accepted. It means that competitiveness in Statistics is 1:12. Meanwhile for Actuarial Science, since this wasa new study program, there were 847 applicants, but only 27 students were accepted. The competitiveness in Actuarial Science was higher since it only accepted 27 out of 847 applicants, meaning that competitiveness was 1:31, or we can say only 1 of 31 applicantswas accepted in Actuarial Science study program. The score can be seen in the summary presented in Figure 2.



The boxplot in Figure 2 shows us the registered students' scores (applicants) based on the status in each study program. We can see that some boxplots are overlapping one another, because there were, at the time, no clear criteria for student acceptance. Aside from that, we can also see that there are so many outliers located above the median of the data. From the outlier, we can see that some of accepted students hada lower score than the unaccepted ones. This is why we tried to model this using Bernoulli to have students with high competence in their chosen study program. In addition, this research would only focus in modeling the three study programs, namely mathematics, statistics and actuarial science.

4.3. Parameter Estimation and Formed Model

The parameters were estimated using OpenBUGS. Based on the model shown in Equation (7), we can make a doodle structured from each node in every Equation (7). The doodle for this research is shown in Figure 3. The nodes in the mixture data are shown by *phi* [i], which are stochastic nodes with Bernoulli, bern[1, i] and bern[2, i]. The node require a prior distribution, symbolized by P, which has Dirichlet distribution. The process of parameter estimation was done iteratively 10000 times with a thin of 600. This iterative process is called the Gibbs Sampler. This process would be carried out in each study program so that it would be easy to determine the BMM model that is formed.after estimation parameter finished, the model formed will be used for evaluate the new student acceptance status.

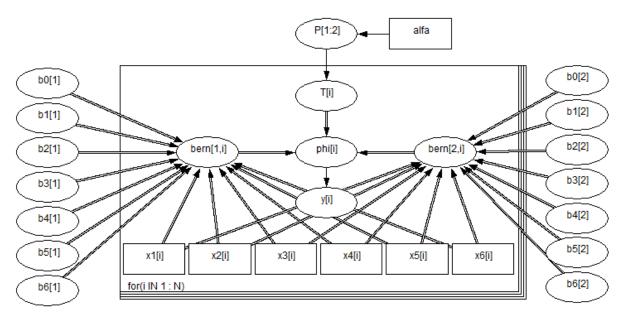


Figure 3. Boxplot for Registered Student' Scores

4.3.1. Parameter Estimation for Mathematics Study Program

For mathematics study program, the parameters are significant in Mixture 1 consisting of x_2, x_5, x_6 . Meanwhile, in Mixture 2, the significant parameters are $x_2 - x_6$. Table 4 show the estimated parameters for Mathematic study program. Based on the Equation (7), we can get the BMM for mathematics study program as follows:

$$P_{mix}(\pi, \mathbf{x}, \mathbf{\beta}) = 0.6535 \ \frac{a}{1+a} + \ 0.3465 \ \frac{b}{1+b}$$

With $a = \exp(-16.32 + 0.0074x_2 + 0.0111x_5 + 0.0066x_6)$ and $b = (-34.74 + 0.0120x_2 + 0.0221x_3 + 0.0062x_4 + 0.0062x_5 + 0.0096x_6)$

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Parameter	Mean	Deviation Standard	2.50%	median	97.50%	Significant
b1[1]	0.0060	0.0063	-0.0064	0.0060	0.0185	No
b2 [1]	0.0073	0.0037	0.0001	0.0073	0.0148	Yes
b3[1]	-0.0024	0.0032	-0.0086	-0.0025	0.0040	No
b4 [1]	-0.0019	0.0024	-0.0067	-0.0019	0.0027	No
b5[1]	0.0111	0.0034	0.0045	0.0110	0.0179	Yes
b6[1]	0.0066	0.0018	0.0030	0.0066	0.0102	Yes
b0[1]	-16.19	3.3080	-22.76	-16.1500	-9.6950	Yes
P [1]	0.6535	0.0215	0.6108	0.6535	0.6957	Yes
b1[2]	0.0000	0.0023	-0.0045	0.0000	0.0045	No
b2[2]	0.0120	0.0030	0.0062	0.0120	0.0178	Yes
b3[2]	0.0221	0.0034	0.0155	0.0221	0.0287	Yes
b4[2]	0.0062	0.0016	0.0030	0.0062	0.0095	Yes
b5[2]	0.0062	0.0020	0.0024	0.0062	0.0102	Yes
b6[2]	0.0096	0.0018	0.0061	0.0095	0.0131	Yes
b0[2]	-34.69	3.4080	-41.45	-34.6800	-27.86	Yes
P[2]	0.3465	0.0215	0.3043	0.3465	0.3892	Yes

Table 4. Estimated Parameters for Mathematic Study Program

4.3.2. Parameter Estimation for Statistics Study Program

For Mathematics study program, the parameters are significant in Mixture 1 consisting of x_2, x_5, x_6 . Meanwhile, in Mixture 2, the significant parameters are $x_2 - x_6$. Table 5 show the estimated parameters for Statistics study program.

Parameter	Mean	Deviation Standard	2.50%	median	97.50%	Significant
b1[1]	0.0322	0.0117	0.0092	0.0323	0.0553	Yes
b2 [1]	0.0040	0.0051	-0.0060	0.0040	0.0143	No
b3[1]	0.0145	0.0053	0.0043	0.0144	0.0251	Yes
b4[1]	0.0093	0.0029	0.0038	0.0093	0.0150	Yes
b5[1]	0.0117	0.0046	0.0028	0.0117	0.0208	Yes
b6[1]	0.0025	0.0025	-0.0024	0.0025	0.0073	No
b0[1]	-42.06	5.7420	-53.34	-41.9600	-31.07	Yes
P [1]	0.7453	0.0189	0.7072	0.7456	0.7811	Yes
b1[2]	0.0203	0.0041	0.0123	0.0202	0.0285	Yes
b2[2]	0.0186	0.0039	0.0111	0.0185	0.0264	Yes
b3[2]	0.0116	0.0030	0.0057	0.0115	0.0176	Yes
b4[2]	0.0069	0.0017	0.0037	0.0069	0.0103	Yes
b5[2]	0.0077	0.0024	0.0030	0.0077	0.0124	Yes
b6[2]	0.0070	0.0017	0.0036	0.0070	0.0103	Yes
b0[2]	-45.44	4.5370	-54.65	-45.3000	-36.86	Yes
P[2]	0.2547	0.0189	0.2190	0.2544	0.2928	Yes

 Table 5. Estimated Parameters for Statistics Study Program

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Based on the Equation (7) we can get he BMM for Statistics study program as follows:

$$P_{mix}(\pi, \mathbf{x}, \mathbf{\beta}) = 0.7453 \ \frac{a}{1+a} + \ 0.2547 \ \frac{b}{1+b}$$

With $a = \exp(-42.31 + 0.0322x_1 + 0.0146x_3 + 0.0094x_4 + 0.0118x_5)$ and $b = \exp(-45.44 + 0.0204x_1 + 0.0185x_2 + 0.0116x_3 + 0.0069x_4 + 0.0077x_5 + 0.0070x_6)$

4.3.3. Parameter Estimation for Actuarial Science Study Program

For Actuarial Science study program, Table 6 show the significant parameters. In mixture 1, consisting of x_6 only, because the competitiveness in this study program is high. Meanwhile, in Mixture 2, the significant parameters are $x_1 - x_6$. Finally, based on the Equation (7), we can get the BMM for Actuarial Science study program as follows:

$$P_{mix}(\pi, \mathbf{x}, \mathbf{\beta}) = 0.7112 \ \frac{a}{1+a} + \ 0.2888 \ \frac{b}{1+b}$$

With $a = \exp(-39.77 + 0.0066x_6)$ and $b = \exp(-44.40 + 0.0224x_1 + 0.0075x_2 + 0.0152x_3 + 0.0045x_4 + 0.0087x_5 + 0.0101x_6)$

Table 6. Estimated Parameters for Actuarial Science Study Program

Parameter	Mean	Deviation Standard	2.50%	median	97.50%	Significant
b1[1]	0.0198	0.0160	-0.0117	0.0197	0.0510	No
b2[1]	0.0096	0.0084	-0.0065	0.0094	0.0267	No
b3[1]	0.0032	0.0068	-0.0093	0.0029	0.0171	No
b4 [1]	0.0059	0.0038	-0.0013	0.0058	0.0136	No
b5[1]	0.0071	0.0072	-0.0069	0.0069	0.0220	No
b6[1]	0.0067	0.0030	0.0008	0.0067	0.0127	Yes
b0[1]	-39.87	7.7930	-56.03	-39.53	-25.33	Yes
P [1]	0.7112	0.0157	0.6798	0.7115	0.7413	Yes
b1[2]	0.0225	0.0036	0.0154	0.0225	0.0296	Yes
b2[2]	0.0074	0.0034	0.0006	0.0074	0.0142	Yes
b3[2]	0.0152	0.0039	0.0077	0.0152	0.0228	Yes
b4[2]	0.0045	0.0017	0.0012	0.0045	0.0079	Yes
b5[2]	0.0087	0.0029	0.0031	0.0087	0.0146	Yes
b6[2]	0.0101	0.0018	0.0067	0.0101	0.0135	Yes
b0[2]	-44.44	4.5420	-53.21	-44.49	-35.49	Yes
P[2]	0.2888	0.0157	0.2587	0.2885	0.3202	Yes

4.3.4. Classification Result of BMM

The model formed in this research will be used for calculating the classifications accuracy in order to evaluate the student acceptance status in ITS, especially for Statistics, Mathematics and Actuarial Sciences. Based on the model formed, the results of classification show in Table 7. The classifications accuracy percentage shows comparison between the actual data of new student acceptance status and the predicted result of model formed. The higher the accuracy percentage, the more accurate model predicted the new student acceptance status. Based on the Table 7 the accuracy given by BMM for Statistics study program is 86.79%, as

for Mathematics study program is 86.46% and the highest accuracy given by Actuarial science study program is 91.77%

 Table 7. Classification Results of BMM

Study Program	% Accuracy
Statistics	86.79
Mathematics	86.46
Actuarial Sciences	91.77

5. Conclusion

In this paper, we have discussed about the process in modeling Bernoulli Mixture Model for educational data. As we can see from the discussion above, it can be concluded that BMM coupled with MCMC has successfully modeled the data and evaluate the new student acceptance status with high accuracy (more than 85%). Higher accuracy means that evaluation for the new student acceptance using the model formed will give small misclassification. However, here's some notes can be a huge help for the further researches. Mixtures can be created based on the will of the researchers or requirements that need to be considered [9].Some methods can be chosenfor parameter estimation, such as MCMC (used in this research), EM algorithm, Variational Bayes methods. If there is sparse dataset, the EM algorithm method can be used to estimate the parameters since it gives results more robust than others [8]. Preprocessing data should be done as the process of initializing and determining which variable will betaken as a mixture. Further, the result of the modeling can be tested by classifying the accepted and unaccepted students and then comparing the classification accuracy (using in-sample and out-sample data), orprobably comparing with another method. Finally, wehope thatthis research could be inspiration for next data classification using Bernoulli Mixture Model.

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References

- [1] McLachlan G, Peel D and Prado P 1998*Clustering Via Normal Mixture Models* (Australia: University of Queensland).
- [2] Celeux G and Govaert G 1991 Clustering criteria for discrete data and latent class models *Journal of Classification*p 157-176.
- [3] Wang X and Kaban A 2005 Finding Uninformative Features in Binary and Automated Learning*Intelligent Data Engineeringp* 40-47.
- [4] Grim J, Somol P and Pudil P 2000 Multivariate structural Bernoulli mixtures for RecognitionProceedings 15th International Conference on Pattern Recognition p585-589
- [5] Bishop C2006Pattern Recognition and Machine Learning, Information Science and Statistics Springer.
- [6] L Abel, J L Golmard and Mallet A 1993 An autologistic model for the genetic analysis of familial binary data*American journal of human genetics* 53(4)p 894-907.
- [7] Ferreira F, Santos V and Dias J 2006 Robust Place Recognition Within Multi-Sensor View Sequences Using Bernoulli Mixture Models *IFAC Proceedings Volumes 40(15)* p 529:534.
- [8] Ye M, Zhang P and Nie L 2018 Clustering sparse binary data with hierarchical Bayesian Bernoulli mixture model *Computational Statistics & Data Analysis Volumes 123* p 32-49.
- [9] Suryaningtyas W, Iriawan N, Ulama B, Fithriasari K, Susanto I and Pravitasari A 2018 On The Bernoulli Mixture Model for Bidikmisi Scholarship *Journal of Physics: Conference Series.*

- [10] Saeed M, Javed K and Babri H A 2013 Machine learning using Bernoulli mixture models : Clustering, Rule, Extraction and Dimensionality Reduction *Neurocomputing119*p 366-374.
- [11] McCullagh P and Nelder J A 1989*Generalized Linear Models, Second Edition* Boca Raton: Chapman and Hall/CRC press.
- [12] McLachlan G and Peel D 2000 Finite Mixture Models New York: John Wiley and Sons Inc.
- [13] Nadif M and Govaert G 1998 Clustering For Binary Data And Mixture Models Choice Of The ModelApplied Stochastic Models and Data Analysisp 269-278.
- [14] Geman S and Geman D 1984 Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images*IEEE Explore : Transactions on Pattern Analysis and Machine Intelligence*.
- [15] Ntzoufras I 2009 Bayesian Modeling Using Win Bugs New York: John Wiley and Sons Inc.